Novel Model for Uniaxial Strain-Rate–Dependent Stress–Strain Behavior of Ethylene–Propylene–Diene Monomer Rubber in Compression or Tension

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ABSTRACT: Based on quasi-static and dynamic experimental results, a novel strain-rate–dependent model for ethylene–propylene–diene monomer (EPDM) rubber was developed. This model, composed of a base model and rate-sensitivity terms, has a relatively simple form to be embedded in computer codes for numerical simulations. The base model combines a Maxwell model and a Mooney function. The Maxwell model is necessary to describe small-strain behavior, whereas the Mooney function dominates the large-strain behavior. Each of these two components is then multiplied by a rate-sensitive term to describe the material's strain-rate sensitivities at both small and large strains. This model gives a good description of EPDM response in both compression and tension over a wide range of strain rates with a minimum number of material constants. © 2004 Wiley Periodicals, Inc. J Appl Polym Sci 92: 1553–1558, 2004

Key words: ethylene–propylene–diene monomer (EPDM); mechanical properties; strain rate; compression; tension

INTRODUCTION

Ethylene–propylene–diene monomer (EPDM) is a crosslinked polymer that has been one of commonly used industrial polymers because of its outstanding resistance to ageing from heat, light, oxygen, and ozone.[1] This rubber has also been widely used as shock absorbers in automotive, aerospace, and portable electronics applications. It is necessary to understand and quantitatively describe its mechanical properties at various strain rates under both compressive and tensile loading conditions such that numerical simulations of structural response to shock loading can be conducted for design optimization.[2] Because of the long history of the application of rubber materials in industry, there are numerous models based on strain-energy functions for rubbers under quasi-static loading conditions.[3–6] However, few efforts have been focused on modeling their behavior with strain-rate effects, although rubbers are known to be strongly sensitive to strain rates and are subjected to shock loadings.

Empirical models, such as a modified Johnson–Cook model and viscoelastic/viscoplastic constitutive models, have also been used to describe the mechanical properties of polymers at various strain rates because of their simplicity, even though they may lack the support of physical mechanisms.[7,8] Bergström and Boyce[9,10] recently used two interacting macromolecular networks to construct a three-dimensional constitutive model that accounted for rate dependency: the first network, called the equilibrium network, is represented by the Arruda–Boyce eight-chain model of rubber elasticity, and the second network is represented by another eight-chain network with a relaxed configuration. This constitutive model is applicable to a wide strain-rate range in principle, as long as the assumptions on the mechanisms (eight-chain networks) are valid. However, the large number of material constants in this model are difficult to be uniquely determined by experiments. A nonlinear viscoelastic constitutive model based on the assumptions of nonlinear elasticity and linear viscoelasticity has recently been developed.[11] The basic formulation is based on stress relaxation functions with two different relaxation times to enable the model to describe strain-rate effects of polymers. However, this model cannot describe the mechanical behavior of solid polymers at large strains. As mentioned earlier, strain-energy functions have been commonly used to describe mechanical properties of rubber or rubberlike materials under quasi-static loading conditions. When strain-rate effects are accounted for, a model purely based on strain-energy functions will become very complicated because of the rapidly increasing number of material constants corresponding to strain-rate effects. Song and Chen[12] presented a relatively simple dynamic model for EPDM rubber with fewer material constants. However, the application range of this model is limited

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to their experimental conditions. Using a strain-energy function and a stress-relaxation function with only one relaxation time, Yang et al.[13] presented a viscohyperelastic model that combines static hyperelastic behavior and a viscoelastic model for incompressible rubberlike materials. This model provides a good description of compressive behavior for SHA rubber at large strains (>0.2), but is not accurate in describing small-strain behavior. Because of the lack of proper rate-dependent material models, quasi-static models that do not account for strain-rate effects have been used in dynamic simulation codes, such as DYNA3D,[14] to describe rubber responses under impact conditions. This indicates the strong need for accurate material models covering a wide range from quasi-static to dynamic strain rates for rubbers, which have relatively simple forms to be used for numerical simulations. Given that shock and vibration loads on the rubber component can be tensile or compressive, or both, it is desirable to develop a model that describes the strain-rate–dependent mechanical behaviors of rubbers under both compressive and tensile loading conditions.

Based on our experimental research on an EPDM rubber in both compression and tension under both quasi-static and dynamic loading,[12,15] a new rate-dependent material model was developed in this study to describe both compressive and tensile properties of the rubber. The seven independent material constants were determined by compressive and tensile experiments at various strain rates from 10^{-3}/s to 10^3/s.

MODELING OF UNIAXIAL STRAIN-RATE–DEPENDENT STRESS–STRAIN BEHAVIOR

Both compressive and tensile stress–strain behaviors of EPDM rubber were previously determined to be strain-rate sensitive and nonlinear.[12,15] An empirical form of a material model typically takes the form

$$\sigma = f(\varepsilon)g(\dot{\varepsilon}) \tag{1}$$

where the term $f(\epsilon)$ represents the strain-rate independent behavior, whereas another term $g(\dot{\varepsilon})$ accounts for the effects of strain rate. This simple formulation can describe only the strain-rate sensitivity independent of the strain in the material, which is not consistent with the experimental results for the specific EPDM rubber under investigation.[12,15] A more general strain-rate–sensitive model may be expressed as

$$\sigma = \sum_{i=1}^{n} f_i(\varepsilon)g_i(\dot{\varepsilon}) \tag{2}$$

where $f_i(\epsilon)$ and $g_i(\dot{\varepsilon})$ represent strain-rate–independent behaviors and strain-rate effects at different strain levels. We seek the simplest formulation of eq. (2) in this research to accurately represent the EPDM behavior determined in experiments.[12,15] First, we find an expression to describe the strain-rate–independent behavior at some reference strain rate. Strain-rate effects terms will be added on later. For the description of the reference behavior at very large strains at a specific strain rate, strain-energy functions have been successfully used.[4] We also use a strain-energy function to describe the large-strain response of EPDM at a reference strain rate. With the assumption of incompressible volume, the strain-energy function approach of the stress–strain relationship for a rubber material under one-dimensional stress can be expressed as[4]

$$\sigma = 2\left(\lambda^2 - \frac{1}{\lambda}\right)\left(\frac{\partial U}{\partial I_1} + \frac{1}{\lambda}\frac{\partial U}{\partial I_2}\right) \tag{3}$$

where λ is the stretch ratio; I_1 and I_2 are strain invariants, defined as

$$I_1 = \lambda^2 + \frac{2}{\lambda} \quad I_2 = \frac{1}{\lambda^2} + 2\lambda \tag{4}$$

and U is the strain-energy function, which can generally be written as

$$U = \sum_{i=0,\, j=0}^{\infty} C_{ij}(I_1 - 3)^i(I_2 - 3)^i \tag{5}$$

where C_{ij} are polynomial coefficients with $C_{00} = 0$ because strain energy disappears at zero strain; and i and j are polynomial powers. It is noted that the stress in eq. (3) refers to true stress. The most general first-order relationship for U of an incompressible material is

$$U = C_1(I_1 - 3) + C_2(I_2 - 3) \tag{6}$$

where C_1 and C_2 are constants. This form of the strain-energy function, called the Mooney equation, was first derived by Mooney on the assumption that a linear stress–strain relationship existed in shear, and has been accepted by many researchers.[5,6] Substitution of eq. (6) into eq. (3) yields a stress–stretch model based on a strain-energy function

$$\sigma = 2\left(\lambda^2 - \frac{1}{\lambda}\right)\left(C_1 + \frac{1}{\lambda}C_2\right) = 2\left[C_1\left(\lambda^2 - \frac{1}{\lambda}\right) + C_2\left(\lambda - \frac{1}{\lambda^2}\right)\right] \tag{7}$$

which can describe the hyperelastic behavior of EPDM rubber at large strains well. However, such a simple model is not accurate to describe the EPDM behavior at small strains, even after an additional term is adopted.[12]

The model accuracy at small strains may be improved by including many more terms in eq. (5). However, this approach quickly becomes impractical when a large number of material constants are encountered.

To keep the model simple and the number of constants small, we adopted a viscoelastic model to describe the material behavior at small strains.[11] A general constitutive behavior for a linear viscoelastic material can be expressed as[16]

$$\sigma(t) = \int_{-\infty}^{t} \phi(t-\tau)\,\frac{d\varepsilon(\tau)}{d\tau}\,d\tau \tag{8}$$

where $\phi(t)$ is a stress-relaxation function, expressed as

$$\phi(t) = \sum_{i=1}^{N} E_i \exp(-t/\theta_i) \tag{9}$$

Equation (9) represents the stress-relaxation behavior of a viscoelastic material, which consists of N Maxwell components. Adopting the first two terms in eq. (9) in addition to a nonlinear elastic component leads to the constitutive model developed by Wang and associates,[11,16] which well described small-strain behavior for polymers. In eq. (9), E_i represents the modulus of spring component in the Maxwell component numbered i; and θ_i is the relaxation time of this component:

$$\theta_i = \frac{\eta_i}{E_i} \tag{10}$$

where η_i is the viscosity of dashpot in the Maxwell component. When θ_i is much less than a deformation time t, the stress-relaxation function $\phi(t)$ will approach zero [eq. (9)]. When the material is loaded over a very wide strain-rate range, the relaxation time θ_i may depend on the strain rate for certain materials. We have attempted to use only one equivalent rate-dependent relaxation time Θ to describe the EPDM small-strain behavior and found that such a simple formulation was capable of accurately capturing the material responses at small strains. This equivalent relaxation time Θ increases with deformation time t, or decreases with strain rate at some fixed strain. Under the condition of constant strain rate, a constant equivalent strain ϵ_r is proposed to describe the relationship between the equivalent relaxation time and strain rate:

$$\Theta = \frac{\varepsilon_r}{\dot{\varepsilon}} \tag{11}$$

With this new variable, eq. (9) is simplified to

$$\phi = Ee^{-(t/\Theta)} = Ee^{-(\varepsilon/\varepsilon_r)} \tag{12}$$

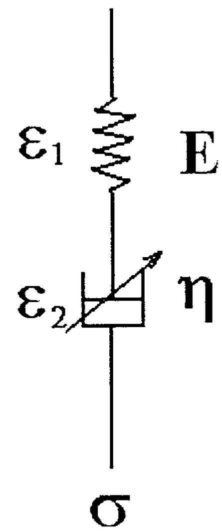

Figure 1 A Maxwell model with a nonlinear dashpot.

where E is the equivalent modulus of the spring components. By substituting eq. (12) into eq. (8) we obtain

$$\sigma = E\varepsilon_r(1 - e^{-(\varepsilon/\varepsilon_r)}) = \sigma_r(1 - e^{-(\varepsilon/\varepsilon_r)}) \tag{13}$$

where $\sigma_r = E\varepsilon_r$.

Equation (13) may also be derived from a nonlinear Maxwell model with a spring (E) and a dashpot ($\eta = \sigma_r/\dot{\varepsilon}$), as shown in Figure 1. Although the strain-rate effects disappear in eq. (13), this equation is found to well describe stress–strain behavior of the EPDM rubber at small strains.

By combining eqs. (7) and (13), a strain-rate independent stress–strain behavior of EPDM rubber at both small and large strains can be expressed with three terms as follows:

$$\sigma = 2C_1\left(\lambda^2 - \frac{1}{\lambda}\right) + 2C_2\left(\lambda - \frac{1}{\lambda^2}\right) + \sigma_r(1 - e^{-(\varepsilon/\varepsilon_r)}) \tag{14}$$

which can also be represented by an illustrative model as shown in Figure 2. Equation (14) represents the base model for the EPDM rubber, which is composed of a simple strain-energy function (first two terms on the right) and a modified Maxwell model (the last term). The modified Maxwell component describes the stress–strain behavior at small strains and will be a constant at large strains where the strain-energy function dominates.

We now seek an effective approach to build strain-rate effects in the model. As mentioned earlier, the strain-rate sensitivity for EPDM rubber varies with strain.[12,15] The model should be capable of describing strain-rate effects as a function of strain. After cross-examining eq. (14) and our experimental results for

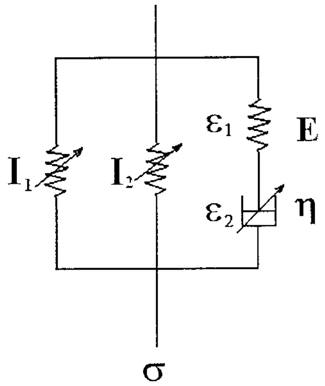

Figure 2 Strain-rate–independent base model.

the EPDM rubber in either tension or compression, it was discovered that the first term in eq. (14) was strain-rate independent, whereas the last two terms did depend on strain rate. Therefore the strain-rate–dependent constitutive model for the EPDM can be expressed as

$$\sigma = D_0\left(\lambda^2 - \frac{1}{\lambda}\right) + f_1(\dot{\varepsilon})\left(\lambda - \frac{1}{\lambda^2}\right) + f_2(\dot{\varepsilon})(1 - e^{-(\varepsilon/\varepsilon_r)}) \quad (15)$$

where $D_0 = 2C_1$; $f_1(\dot{\varepsilon})$and $f_2(\dot{\varepsilon})$ correspond to strain-rate effects at large strains and at small strains, respectively, because the term of the second strain invariant describes the large-strain behavior and the term of the Maxwell component describes the small-strain behavior. We used an exponential formulation for the rate sensitivity terms. Such formulations have recently been used to describe the strain-rate effects of materials including metals[17] and composites.[18]

$$f(\dot{\varepsilon}) = a + b\dot{\varepsilon}^{\alpha} \quad (16a)$$

or

$$f(\dot{\varepsilon}) = a + b_1\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{\alpha} \quad (16b)$$

where $\dot{\varepsilon}_0$ is the reference strain rate; α, a, and b are material constants; and

$$b_1 = b\dot{\varepsilon}_0^{\alpha}$$

Now, the strain-rate–dependent model for the EPDM rubber can be expressed as

$$\sigma = D_0\left[(1+\varepsilon)^2 - \frac{1}{1+\varepsilon}\right] + \left[A_0 + A_1\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{\alpha_1}\right] \times \left[(1+\varepsilon) - \frac{1}{(1+\varepsilon)^2}\right] + \left[B_0 + B_1\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{\alpha_2}\right] \times (1 - e^{-(\varepsilon/\varepsilon_r)}) \quad (17)$$

where A_0, A_1, B_0, and B_1 are material constants to be determined experimentally.

DETERMINATION OF MATERIAL CONSTANTS FOR EPDM RUBBER

Stress–strain curves at various strain rates for EPDM rubber were previously obtained under valid compressive and tensile testing conditions.[12,15] Because the mechanical responses for EPDM rubber in compression are different from those in tension, some of the material constants in the model [eq. (17)] for compression and for tension cases may also be different. In eq. (17), the stress is true stress, whereas the strain is engineering strain. The engineering strain is negative in compression and positive in tension.

The material constants for eq. (17) determined using experimental results[12,15] are tabulated in Table I. The material constants in compression are different from those in tension. When the EPDM rubber is in compression, the first term in eq. (17) disappears ($D_0 = 0$ in compression). However, when the EPDM rubber is in tension, the last term in eq. (17) is linearly sensitive to the strain rate. The material model [eq. (17)] can thus be expressed with seven independent material constants as follows:

In compression:

$$\sigma = \left[A_0 + A_1\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{\alpha_1}\right]\left[(1+\varepsilon) - \frac{1}{(1+\varepsilon)^2}\right] + \left[B_0 + B_1\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{\alpha_2}\right](1 - e^{-(\varepsilon/\varepsilon_r)}) \quad (18a)$$

In tension:

$$\sigma = D_0\left[(1+\varepsilon)^2 - \frac{1}{1+\varepsilon}\right] + \left[A_0 + A_1\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{\alpha_1}\right] \times \left[(1+\varepsilon) - \frac{1}{(1+\varepsilon)^2}\right] + \left[B_0 + B_1\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\right](1 - e^{-(\varepsilon/\varepsilon_r)}) \quad (18b)$$

TABLE I
Material Constants in Eq. (18) for EPDM Rubber in Compression and Tension

Constant	Compression	Tension
D_0	0	4.0
A_0	0.39	−8.0
A_1	7.1425×10^{-7}	0.06269
B_0	−0.20	3.30
B_1	-1.7271×10^{-3}	-6.32×10^{-6}
α_1	1.0614	0.3502
α_2	0.4631	1
ε_r	−0.02	0.18
$\dot{\varepsilon}_0$	0.0015	0.004

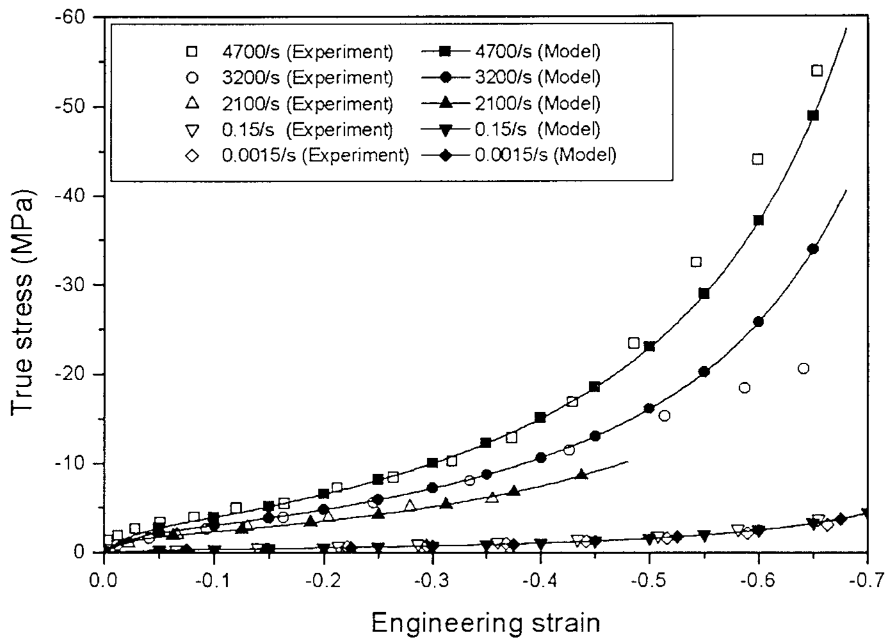

Figure 3 Comparison of compressive stress–strain curves of EPDM rubber from experiments and from the model at various strain rates.

Figures 3 and 4 show the comparisons of stress–strain curves at various strain rates by experiments and by the model under compressive and tensile loadings, respectively. The good agreements between the model descriptions and the experimental results under both compressive and tensile loadings (Figs. 3 and 4) indicate that the model developed in this study is capable of accurately describing the strain-rate–dependent mechanical behaviors of the EPDM rubber under both compressive and tensile loading conditions at both quasi-static and dynamic loading rates. The relatively small number of material constants and the relatively simple formulation make the constitutive model very efficient for application in numerical simulations.

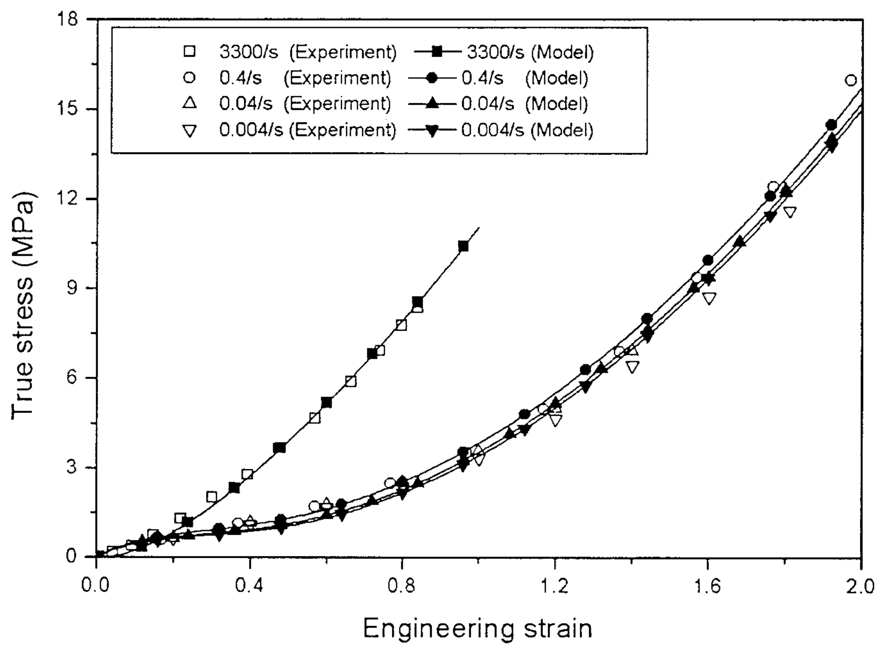

Figure 4 Comparison of tensile stress–strain curves of EPDM rubber from experiments and from the model at various strain rates.

CONCLUSIONS

A new strain-rate–dependent model for EPDM rubber is developed in this study. A strain-rate independent base model is constructed first, which consists of a hyperelastic component derived from Mooney equation to describe the stress–strain behavior at large strains, and a modified nonlinear Maxwell component, which describes the small-strain behavior. This unique combination decreases the number of material constants without compromising model accuracy. Based on experimental results on EPDM rubber under both compressive and tensile loading conditions, two exponential strain-rate sensitivity terms are embedded in the base model to describe the strain-rate dependencies at both large and small strains. The model exhibits good agreement with the experimental results over wide ranges of strain rates, under both compressive and tensile loading conditions. The relatively small number of material constants and simple formulation increase the applicability of the model in numerical simulations.

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